The Plurality-with-Elimination Method

Lecture 9
Section 1.4

Robb T. Koether

Hampden-Sydney College

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Outline

The Plurality-with-Elimination Method

Definition (The Plurality-with-Elimination Method)

By the plurality-with-elimination method (also called instant-runoff voting, or IRV),

- The voters cast their votes for their *first-place* choice.
- If one candidate has a majority of votes, he wins.
- Otherwise, the candidate with the *fewest first-place* votes is eliminated and the process repeats with the remaining candidates until there is a winner.

The Political Science Club Election

Example (The Political Science Club Election)

	15	11	6	3
1st	A	B	C	C
2nd	D	A	D	B
3rd	C	D	B	A
4th	B	C	A	D

- Who is the winner?
- Give the complete ranking (in reverse order of elimination).

The Political Science Club Election

Example (The Political Science Club Election)

	15	11	6	3
1st	A	B	B	A
2nd	D	A	D	B
3rd	C	D	C	C
4th	B	C	A	D

- What if there is a tie (C and D each received 0 first-place votes)?
- Which one do we eliminate?

The Political Science Club Election

Example (The Political Science Club Election)

	15	11	6	3
1st	A	B	B	A
2nd	D	A	D	B
3rd	C	D	C	C
4th	B	C	A	D

- What if there is a tie (C and D each received 0 first-place votes)?
- Which one do we eliminate?
- Does it matter?

What if There is a Tie?

Example (What if There is a Tie?)

	10	8	8	8
1st	A	B	C	D
2nd	B	C	D	B
3rd	C	D	B	C
4th	D	A	A	A

- Eliminate B. Who wins?

What if There is a Tie?

Example (What if There is a Tie?)

	10	8	8	8
1st	A	B	C	D
2nd	B	C	D	B
3rd	C	D	B	C
4th	D	A	A	A

- Eliminate B. Who wins?
- Eliminate C. Who wins?

What if There is a Tie?

Example (What if There is a Tie?)

	10	8	8	8
1st	A	B	C	D
2nd	B	C	D	B
3rd	C	D	B	C
4th	D	A	A	A

- Eliminate B. Who wins?
- Eliminate C. Who wins?
- Eliminate D. Who wins?

What if There is a Tie?

Example (What if There is a Tie?)

	10	8	8	8
1st	A	B	C	D
2nd	B	C	D	B
3rd	C	D	B	C
4th	D	A	A	A

- Eliminate B. Who wins?
- Eliminate C. Who wins?
- Eliminate D. Who wins?
- Let's not worry about that.

Outline

Faster Elimination

Rather than eliminate the candidates one per round, we could eliminate

- Two per round (or three, or four, etc.)
- All but two in the first round.

Example

Example

Suppose that there are 5 candidates: A, B, C, D, E. The following table summarizes the voters' preferences.

No. of voters	Preferences						
	6	4	4	4	3	1	1
1st	B	B	D	E	A	C	C
2nd	A	A	A	C	D	B	D
3rd	C	D	E	D	C	A	A
4th	D	E	C	B	B	D	B
5th	E	C	B	A	E	E	E

- Use the elimination method, 2 at a time, to find the winner.
- Would the result be the same if we eliminated them one at a time?

Outline

A Defect

	7	8	10	4
1st	A	B	C	A
2nd	B	C	A	C
3rd	C	A	B	B

- What could possibly go wrong with this method?
- Who is the winner?

A Defect

	7	8	10	4
1st	A	B	C	A
2nd	B	C	A	C
3rd	C	A	B	B

- What could possibly go wrong with this method?
- Who is the winner?
- What if the 4 voters who preferred A over C (in the last column) changed their minds and preferred C over A.
-
-

A Defect

	7	8	10	4
1st	A	B	C	C
2nd	B	C	A	A
3rd	C	A	B	B

- What could possibly go wrong with this method?
- Who is the winner?
- What if the 4 voters who preferred A over C (in the last column) changed their minds and preferred C over A.

A Defect

	7	8	10	4
1st	A	B	C	C
2nd	B	C	A	A
3rd	C	A	B	B

- What could possibly go wrong with this method?
- Who is the winner?
- What if the 4 voters who preferred A over C (in the last column) changed their minds and preferred C over A.
- That could only help C, right?

A Defect

	7	8	10	4
1st	A	B	C	C
2nd	B	C	A	A
3rd	C	A	B	B

- What could possibly go wrong with this method?
- Who is the winner?
- What if the 4 voters who preferred A over C (in the last column) changed their minds and preferred C over A.
- That could only help C, right?
- Wrong!

Outline

Coombs' Method

Definition (Coombs' Method)

Coombs' method is a variation of the plurality-with-elimination method. The voters cast their votes for their *last-place* choice. The candidate with the *most last-place* votes is eliminated and the process repeats with the remaining candidates until there is a winner.

The Political Science Club Election

Example (The Political Science Club Election)

	15	11	6	3
1st	A	B	C	C
2nd	D	A	D	B
3rd	C	D	B	A
4th	B	C	A	D

- Who is the winner?
- Give the complete ranking.
- How do the results compare to the plurality-with-elimination method?

Outline

Assignment

Assignment

- Chapter 1 Exercises 31, 32, 33, 35, 37, 38, 69a.
- Rework 31, 32, and 33 using Coombs' method. Were the results the same as with the Plurality-with-Elimination Method?